winds 3 times around the axial circle. Its equation is easily found to be

$$R^2(x^2+y^2)^3=[(R^2-r^2)(x^2+y^2)+r(3x^2y-y^3)]^2,$$

and is, as we expect, of the 6th order.

Every closed loxodromic has an n-fold symmetry with respect to meridian-planes. Hence the cylinder with its xy-projection as a base cuts the torus in another loxodromic which is the reflection of the first on the xy-plane. The loxodromic is thus generally of order $4n$.

The University of Colorado, November, 1902.

ON BALL'S HISTORY OF MATHEMATICS.

By DR. G. A. MILLER.

Among the few histories of mathematics in the English language the third edition of Ball's work is the latest and most extensive.* It is therefore natural that this work should find a place in the libraries of many teachers of mathematics in these days of deep interest in the history of science. The following corrections and additions may possibly prove helpful especially to those who do not have the time or opportunity to study Cantor's great work.†

On page 4, Ball says, "The Egyptians and Greeks simplified the problem by reducing a fraction to the sum of several fractions, in each of which the numerator was unity, so that they had to consider only various denominators: the sole exceptions being the fractions $\frac{2}{3}$ and $\frac{3}{4}$. This remained the Greek practice until the sixth century of our era."

Being unable to find any instance where the ancient Egyptians had used the fraction $\frac{3}{4}$ I recently wrote Ball about the matter and received the following reply, "I cannot find my authority (if I had one) for the statement that the Egyptians used the fraction $\frac{3}{4}$. I fear it must have been an error which I have repeated in each edition without verification."

The above quotation from Ball's History is also apt to convey a false impression in regard to the Greek methods of dealing with fractions. While it is true that the Greeks employed unit fractions to a considerable extent, yet they also employed fractions with a general numerator and had a general notation for such fractions. In these respects they differed very widely from the Egyptians, although one might be led to infer the contrary not only from the above quotation but also from the statement on page 76.

On page 13 it is stated that the history of mathematics written by

* A Short Account of the History of Mathematics. By W. W. R. Ball, The Macmillan Company, New York, 1901, pp. xxiv+527.

† Vorlesungen ueber Geschichte der Mathematik. Von Moritz Cantor, Teubner, Leipzig, 1884-1901.

Theophrastus has been lost. Although Cantor makes a similar remark on page 108 (Vol. 1) of his history, yet it seems to have been proved quite recently that such a history never existed.* The statement on page 59 that Books VII, VIII, IX, and X of Euclid's Elements are devoted to the theory of numbers is found in each of the three editions. Since Book X is devoted to irrational magnitudes it cannot be classed with theory of numbers according to the common use of this term.

As Archimedes had employed the formula to find the sum of an arithmetical progression and the Pythagoreans had found the sum of certain arithmetical series, the statement on page 87, "Hypsicles developed the theory of arithmetical progressions which had been so strangely neglected by the earlier mathematicians," seems to give too much credit to Hypsicles. Even the ancient Egyptians were acquainted with the formula to find the sum of an arithmetical series, while such series were also known among the Babylonians.

Unfortunately, Ball gives a number of dates as definitely established which are known only approximately. For instance, it is not known exactly when the author of the most influential Greek arithmetic lived, yet Ball states definitely on page 97 that Nicomachus was a Jew, who was born at Gerasa in the year 50. Two pages later he states that Ptolemy died in 168 which is equally uncertain. The famous Neopolitan lad of 16 who is mentioned on page 103 was Annibale Giordano and not Oltajano. His home was in Ottajano.

Among the references on page 125 there should be included the valuable work of Conant, entitled Number Concept, published by *The Macmillan Company.* The tribe of West Africa, mentioned on page 127, counted by multiples of six and not by multiples of seven. On the same page it is stated that the Hindoos used the abacus or swan-pan, while on page 161 we read more correctly that "the Arabs (like the Hindoos) seem also to have made little or no use of the abacus." The fact is that the existence of the abacus among the Hindoos has not yet been established.

In the Roman abaci which I have seen pictured elsewhere, the marginal grooves or wires were used for fractions whose denominators are 12, 24, 36, 48, and 72, and not for those whose denominators are 4, as stated by Ball on page 129. The object which Leonardo had in view in writing his famous Liber Abaci seems to have been entirely misstated on page 174. That the words "in order that the Latin race might no longer be deficient in that knowledge" cannot refer to the Arabic system of notation follows directly from Leonardo's own words. It is quite probable that they refer to the method of "false assumption" upon which so much stress is laid in the Liber Abaci. On page 184 we read that Bradwardine "was the first European to introduce the cotangent into trigonometry." This function had been used earlier by Robertus Anglicus.

Pages 185 and 186 present a very remarkable state of affairs. While Oresme is the greatest French mathematician of the fourteenth century, yet Ball

* Cf. Enestroem, *Bibliotheca Mathematica*, Vol. 3, 1902, p. 246. At this place Enestroem notes a number of errors in Ball's History. Some of the most important one are repeated in the present note.

says "I do not propose to discuss his writings." The Latitudines formarum of Oresme explains how to draw a curve which represents the changes of a function during a given period, just like our modern temperature curves, and exhibits some of the properties of such curves. It indicates a very important step towards analytic geometry and hence it is of great historical interest. It was studied very extensively and exerted a powerful influence on the mathematical teachings of those times. Why such a work should be passed in silence is difficult to see. This becomes the more remarkable if it is observed that Ball gives on page 186 the courses in mathematics offered at the University of Vienna in 1389. One of these courses was devoted to Latitudines but as he failed to explain this subject at its proper place he is compelled to give an incomplete list of these courses. He seems to have substituted "measurement of superficies" for latitudines formarum, which conveys a totally wrong impression.

On page 195 we read, "and the test of the accuracy of the result by casting out the nines was invented by the Arabs." This test seems to have been employed earlier by the Hindoos. Record is believed to have been the first to use the modern sign for equality and gives as his reason for selecting this particular symbol that no two things can be more equal than two parallel straight lines. Notwithstanding this definite statement by Record we are told on page 221 that the symbol $=$ was a recognized abbreviation for the word *est* in medieval manuscripts; and this would seem to indicate a more probable origin than Record's own words.

We shall call attention to only one more statement, which is not incorrect but somewhat misleading. In speaking about Gauss, on page 462 we are told that "he discussed the binomial equation of the form $x^n=1$: this involves the celebrated theorem that it is possible to construct by elementary geometry regular polygons of which the number of sides is $2^m(2^n+1)$, where m and n are integers and 2^n+1 is a prime."

Why should it be $2^m(2^n+1)$ when it might just as well be $3.2^m(2^n+1)$, $5.2^m(2^n+1)$, or $15.2^m(2^n+1)$, since regular polygons of which the number of sides is of any one of these forms can be constructed in a similar way when $n=0$ or $n>2$. In fact these cases when $n=0$ were all known at the time of Euclid while Gauss made the remarkable discovery that n can have any value greater than 2 provided 2^n+1 is prime. Before the time of Gauss not a single polygon for $n>2$ had been constructed by elementary methods.

It may be added that this matter is stated in an unsatisfactory manner in at least two other recent histories of mathematics. From the statement in Cajori's History of Elementary Mathematics page 74 the reader would be likely to infer that it was impossible to construct a regular polygon of 51 sides by elementary methods, while the English editor of Fink's History of Mathematics, page 207, would seem to imply that even the regular pentagon could not be constructed in this way.

These remarks are not intended to throw discredit on the works cited. Few books are accurate in all details and histories of mathematics are no excep-

tion to this rule. In fact from the great variety of subjects treated, the task of their authors is an unusually difficult one. It is therefore important that the reader should be on his guard and utilize all available material instead of relying completely on some one author.

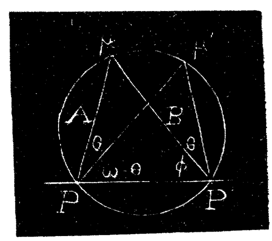
Stanford University, November, 1902.

A DEVELOPMENT OF THE CONIC SECTIONS BY KINEMATIC METHODS.

By JOHN JAMES QUINN, Ph. B., Head of the Department of Mathematics and Manual Training, Warren High School, Warren, Pa.

The Circle.

Proposition I. *If two lines A and B, pivoted at P and P', respectively, be placed at any angle μ to each other, and both revolve in the same direction with the same angular velocity, the locus of their intersection is a circle.*



Let ϕ and ψ denote the angles which A and B in their initial position make with the line PP'. Then

$$\omega+\phi+\mu=180°=\pi.$$

In any new position of A and B,

$$\omega-\theta+\phi+\theta+\mu'=\pi.$$

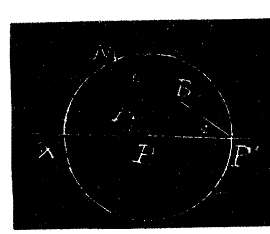
Hence $\mu=\mu'=$constant, so that the locus is a circle.

Remark. If p denotes the distance PP' between the pivots and C denotes the area of the generated circle, then

$$C=\pi p^2,\ \text{if}\ \mu=30°;\quad C=\tfrac{1}{2}\pi p^2,\ \text{if}\ \mu=45°;$$

$$C=\tfrac{1}{3}\pi p^2,\ \text{if}\ \mu=60°;\quad C=\tfrac{1}{4}\pi p^2,\ \text{if}\ \mu=90°.$$

Proposition II. *If two lines A and B, pivoted in an axis X, and initially coincident with it, revolve in the same direction, the one having twice the angular velocity of the other, then the locus of their intersection is a circle.*



Let ϕ be the angle through which B moves in a unit of time; 2ϕ the angle through which A moves in the same time, and θ the vertical angle.

By the conditions, $2\phi=\phi+\theta$. Therefore $\theta=\phi$. Whence the triangle PMP' is isosceles. But the side PP' is constant. Therefore the side PM is constant. Therefore the locus of the intersection of A and B is a circle.